# Afterword

I hope you have enjoyed these forays into mathematics as much as I have, and that you have been left with much to think about. The moral to this story is that beautiful, surprising results are always worthwhile for their own sake, but they can often lead into deeper and even more exciting waters.

While a student at Northern Secondary School in Toronto, Kevin Purbhoo came up with the following beautiful problem:

> On a remote Norwegian mountain top, there is a huge checkerboard, 1000 squares wide and 1000 squares long, surrounded by steep cliffs to the north, south, east, and west. Each square is marked with an arrow pointing in one of the eight compass directions, so (with the possible exception of some squares on the edges) each square has an arrow pointing to one of its eight nearest neighbors. The arrows on squares sharing an edge differ by at most 45°. A lemming is placed randomly on one of the squares, and it jumps from square to square following the arrows.
>
> Prove that the poor creature will eventually plunge from a cliff to its death.
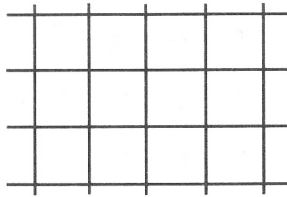
Although Kevin did not know it at the time, this problem anticipates several subtle and important results in topology, a type of geometric thinking useful in most areas of mathematics. This is only one example where a simple idea or problem opens up new mathematical horizons.

If you are a high school student and want to learn more interesting mathematics, there are many avenues you can pursue. First of all, become comfortable writing proofs. Putting ideas to paper in a form convincing to others sharpens the mind; like any new skill, it requires practice. Euclidean geometry provides a perfect opportunity for this, as the tools required are minimal and the rewards great. (Sadly, geometry seems to be quietly disappearing from the high school curriculum.)

The beauty of mathematics has inspired many authors to write expository books. As a result, there are numerous excellent mathematics books aimed at many levels. The *Annotated References* is a good starting point, but there are many other worthwhile books not listed. Browse around, and follow up on authors and subjects you really enjoy. If your school library has only a few titles, convince your librarian to buy more.

Although private reading and independent study are useful, it is important to share your ideas with teachers and other students. A Math Club is a one way to form such a community; arguing about ideas with friends is another. There are also many summer and weekend programs aimed at students; your local university is a good source.

Finally, you might enjoy taking part in mathematical competitions. In Canada, the University of Waterloo runs an excellent series of competitions at many grade levels. (For more information, contact the Canadian Mathematics Competition, Faculty of Mathematics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1, tel. (519) 885-1211 ext. 2248, fax (519) 746-6592.) Success here opens the door to many other competitions, including the *Canadian Mathematical Olympiad* and the *International Mathematical Olympiad*. In both Canada and the U.S., the Mathematical Association of America runs a cycle of competitions beginning with the *American High School Mathematical Examination* (contact: Dr. Walter E Mientka, Executive Director, American Mathematics Competition, 1740 Vine Street, University of Nebraska, Lincoln NE, 68588-0658 USA). Teams from parts of the U.S. and Canada compete every May at the fun and informal *American Regions Mathematics League*. (contact: Mr. Mark E. Saul, ARML President, Bronxville School, Bronxville NY, 10708 USA, (914) 337-5600, e-mail 73047.3156 @compuserve.com). Finally, the *International Mathematics Talent Search* (contact: Dr. George Berzsenyi, Department of Mathematics / Box 121, Rose-Hulman Institute of Technology, Terre Haute IN 47803-3999 USA) is a non-competitive year-long problem-solving program. While most other competitions have stringent time limits, the IMTS allows more reflection on the part of the participants, fostering not only ingenuity, fast thinking, and creativity, but also commitment, reliability, and perseverance.

Once again, I hope you have been challenged and excited by this book. If you have any comments, please write. Best of luck in future explorations!

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