TEAM QUESTIONS

T1. If $\log_2 x^2 + \log_{1/2} x = 5$, compute x.

T2. In triangle ABC, angle $A = 36°$, $AB = AC = 1$, and $BC = x$. If $x = \dfrac{p + \sqrt{q}}{2}$, compute the ordered pair of integers (p,q).

T3. [*Initial setup*: We have 3 jars, called A, B, and C. Jars A and B each contain one white and one black ball; jar C is empty. A ball is now picked at random from jar A (each ball has an equal probability of being picked) and placed into jar C. Similarly, a ball is picked from jar B and placed into jar C.] We now consider jar C. A ball is picked from jar C at random (each ball has an equal probability of being picked), and it is white. The ball is put back into jar C, which is shaken, and again a ball is picked out at random; again it is white. That ball is put back into C, and for a third time a ball is picked at random; again it is white. Compute the probability that the ball still in jar C is black.

T4. In a three-dimensional rectangular coordinate system, triangle ABC has vertices $A(6,0,0)$, $B(0,4,0)$, $C(0,0,5)$. Compute the distance from the centroid of the triangle [the intersection of the medians] to the x-axis.

T5. Complete this "cross-number puzzle" by putting the proper digit into each box. All answers to clues are 3-digit numbers (thus no answer begins with a zero).

1	2	3
4		
5		

ACROSS

1. An integral multiple of the number of ways a standard pair of (6-sided) dice can land
4. A prime number
5. The sum of the entries (numbers) in one of the rows of Pascal's triangle

DOWN

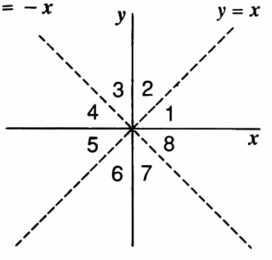
1. The number of entries in one of the rows of Pascal's triangle in which all of the entries are odd
2. An integral multiple of the number of *different-looking* ways a standard pair of (6-sided) dice can land [e.g., 3,4 looks the *same* as 4,3]
3. A perfect square

T6. Let a and b be the lengths of the legs of a right triangle. If a remains constant and b increases continually, which of the following can describe how the ratio of the triangle's area to its perimeter changes [indicate *all* possible answers]?
(1) Increases continually (2) Decreases continually
(3) Remains constant
(4) First increases, then decreases continually
(5) First decreases, then increases continually
(6) Increases and decreases in patterns other than those described in the previous choices

T7. In the coordinate plane, the set of points $A_0, A_1, A_2, A_3, \ldots, A_n$ is determined as follows: A_0 is the origin; A_1 is the point (3,4); A_2 is the image of A_1 reflected through the origin; for $k \geq 3$, A_k is the image of A_{k-1} reflected through A_{k-2}. Compute the length of the line segment $\overline{A_0A_7}$.

T8. If $y = f(x)$ is a continuous *function of x* [that is, a function whose graph has no "breaks" or "jumps"] that passes through the origin but never again touches the lines $y = x$ or $y = -x$ and if $f(f(x)) = x$, then the entire graph (except for the origin) must be in octants r and s. If $1 \leq r < s \leq 8$, name all possible ordered pairs (r,s).

The numbers refer to the eight "octants."



T9. If a, b, c, x, y, and z are real and $a^2 + b^2 + c^2 = 25$, $x^2 + y^2 + z^2 = 36$, and $ax + by + cz = 30$, compute $\dfrac{a + b + c}{x + y + z}$.

T10. From point P outside circle O, tangent segment $\overline{PA}$ and secant $\overline{PBC}$ are drawn. If $PA = OA = 1$ and angle $APC = 75°$, then secant PBC will equal $\dfrac{\sqrt{f} + \sqrt{g}}{2}$. Compute the ordered pair of positive integers (f,g), where $f < g$.