21. Expanding, we get \(1 + 2ab + a^2b^2 + 1 + 2cd + c^2d^2 + a^2c^2 + b^2d^2 = 1 + (ab + cd)^2 + (ac - bd)^2 \geq 1.

22. Let \(M\) and \(N\) denote points of intersection of \(B_2A_2\) with \(BA_1\) and \(AB_1\), respectively (see Figure 6.5). Segment \(B_2N\) is a midline in triangle \(ABB_1\), so \(B_2N = AB/2\), and similarly, \(A_2M = AB/2\). Thus, we have that triangles \(BMA_2\) and \(ANB_2\) are congruent, since they have equal sides \(B_2N = MA_2\), equal angles \(B_2AN\) and \(A_2BM\), and equal altitudes dropped from vertices \(A\) and \(B\), respectively. Consequently, \(\angle AB_2A_2 = \angle BA_2B_2\). Using this equality, we can show in the same way that triangles \(AB_2A_2\) and \(BA_2B_2\) are equal; in particular, \(\angle BB_2A_2 = \angle AA_2B_2\), and therefore \(\angle CA_2B_2 = \angle CB_2A_2\), that is, \(\angle CAB = \angle CBA\), since \(A_2B_2\) is parallel to \(AB\).

![Figure 6.5](image)

23. **Solution 1.** We will show by induction on the number \(N\) of boughs that the total time of the process as well as the number of crows who have flown away does not depend on the order of flights. At the beginning, \(P\) crows are sitting on the highest (\(N\)th) bough of the oak. Consider the process on the other boughs of the oak. If this reduced process takes \(T\) minutes and the number of flown crows is equal to \(Q\), then we can derive that there must be exactly \(P + Q - 1\) crows that have to fly away from the \(N\)th bough (except for the case when \(P = Q = 0\); then there would be no crows of this type). Thus, it takes \(P + Q - 1\) more minutes, and we can conclude that the whole process would take \(T + P + Q - 1\) minutes and that the number of flown crows is equal to \(P + Q - 1\) irrespective of the order of flights.